INTEGRALA NEDEFINITĂ

1. Primitive. Proprietăţi.

Definiţia 1. *Fie f:* I → **R**. *Se spune că f admite primitive pe I dacă* ∃ *F :* I →**R** *astfel încât*
a) F este derivabilă pe I*;*
b) F'(x) =f(x), ∀ *x ε* I.
F se numeşte primitiva lui *f*. (I poate fiinterval sau o reuniune finită disjunctă de intervale).

Teorema 1.1 ***Fie f : I → R. Dacă $F_1, F_2 : I \to \mathbf{R}$ sunt două primitive ale funcţiei f, atunci există o constantă c ∈ R astfel încât $F_1(x) = F_2(x) + c, \forall x \in I$.***

Demonstraţie : Dacă F_1, F_2 sunt primitive atunci F_1, F_2 sunt derivabile $\Rightarrow F_1'(x) = F_2'(x) = f(x)$ ∀ x ε I

$\Leftrightarrow (F_1 - F_2)'(x) = F_1'(x) - F_2'(x) = 0$, x ε I. $\Rightarrow F_1(x) - F_2(x) = c$, c= constantă

OBS 1. Fiind dată o primitivă F_0 a unei funcţii atunci orice primitivă F a lui f are forma F = F_0 + c , c= constantă

$\Rightarrow$ f admite o infinitate de primitive.

OBS 2. Teorema nu mai rămâne adevărată dacă I este o reuniune disjunctă de intervale Expl: f: R-$\{0\}$, f(x) = x²

F = $\frac{x^3}{3}$, G= $\begin{cases} \frac{x^3}{3}+1 \\ \frac{x^3}{3}+2 \end{cases}$ F, G sunt primitive ale lui f dar F-G nu e constantă . Contradicţie cu T 1.1

OBS 3. Orice funcţie care admite primitive are **Proprietatea lui Darboux**.
Se ştie că derivata oricărei funcţii are P. lui Darboux , rezultă că f are P lui Darboux**. F' =f.**

OBS 4. Dacă I este interval şi f(I) $\underset{=}{def} \{f(x) / x \in I\}$ nu este interval atunci f nu admite primitive.

Dacă presupunem că f admite primitive atunci din OBS 3 rezultă că f are P lui Darboux, rezultă f(I) este interval ceea ce este o contradicţie.

OBS 5. Orice funcţie continuă definită pe un interval admite primitive.

Definiţia 2. *Fie f: I →R o funcţie care admite primitive. Mulţimea tuturor primitivelor lui f se numeşte* ***integrala nedefinită*** *a funcţiei f şi se notează prin simbolul* $\int f(x)$ *dx. Operaţia de calculare a primitivelor unei funcţii(care admite primitive) se numeşte* ***integrare***.

Simbolul $\int$ a fost propus pentru prima dată de Leibniz, în 1675.

Fie F(I)= $\{f : I \to R\}$ Pe această mulţime se introduc operaţiile :

1. $(f+g)(x) = f(x) + g(x)$,
2. $(\alpha f)(x) = \alpha . f(x)$ $\forall x \in$ **R** ,α constantă

$\int f(x)\,dx = \{F \in F(I) / F \quad \text{primitivă} \quad a \quad \text{lui} \quad f\}$.

Teorema 1.2 Dacă *f*,*g*:I→ R sunt funcţii care admit primitive şi *α* ∈ R, α ≠0, atunci funcţiile *f*+*g*, *αf* admit de asemenea primitive şi au loc relaţiile:∫(*f*+*g*) =∫*f* +∫*g*, ∫*αf*=*α*∫*f*, *α*≠0, ∫*f* =∫*f* +C

2. PRIMITIVELE FUNCŢIILOR CONTINUE SIMPLE

1. $\boxed{\int c\,dx = c \cdot x + C,\quad c \in R}$ **Ex** $\int 6dx = 6x + C$

2. $\boxed{\int x^n dx = \frac{x^{n+1}}{n+1} + C}$ **Ex.** $\int x^{10} dx = \frac{x^{11}}{11} + C$

3. $\boxed{\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C}$ **Ex** $\int \sqrt[3]{x}dx = \int x^{\frac{1}{3}}dx = \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + C = \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + C = \frac{3}{4}\cdot\sqrt[3]{x^4} + C$

4. $\boxed{\int a^x dx = \frac{a^x}{\ln a} + C}$ **Ex** $\int 2^x dx = \frac{2^x}{\ln 2} + C$

5. $\boxed{\int e^x dx = e^x + C}$ 6. $\boxed{\int \frac{1}{x} dx = \ln|x| + C}$ 7. $\boxed{\int \frac{1}{\sin^2 x} dx = -ctgx + C}$

8. $\boxed{\int \frac{1}{\cos^2 x} dx = tgx + C}$ 9. $\boxed{\int \sin x dx = -\cos x + C}$ 10. $\boxed{\int \cos x dx = \sin x + C}$

11. $\boxed{\int \frac{1}{x^2+a^2}dx = \frac{1}{a} arctg \frac{x}{a} + C}$ **Ex** $\int \frac{1}{x^2+5^2}dx = \frac{1}{5} arctg \frac{x}{5} + C$

12. $\boxed{\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C}$ **Ex** $\int \frac{1}{x^2-25}dx = \frac{1}{10}\ln\left|\frac{x-5}{x+5}\right| + C$

13. $\boxed{\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln(x+\sqrt{a^2+x^2}) + C}$ **Ex** $\int \frac{1}{\sqrt{x^2+4^2}}dx = \ln(x+\sqrt{4^2+x^2}) + C$

14. $\boxed{\int \frac{1}{\sqrt{x^2-a^2}}dx = \ln\left|x+\sqrt{x^2-a^2}\right| + C}$ **Ex** $\int \frac{1}{\sqrt{x^2-49}}dx = \ln\left|x+\sqrt{x^2-49}\right| + C$

15. $\boxed{\int \frac{1}{\sqrt{a^2-x^2}}dx = \arcsin\frac{x}{a} + C}$ **Ex** $\int \frac{1}{\sqrt{16-x^2}}dx = \arcsin\frac{x}{4} + C$

16. $\boxed{\int tgx dx = -\ln|\cos x| + C}$ 17. $\boxed{\int ctgx dx = \ln|\sin x| + C}$

18. $\boxed{\int \frac{x}{\sqrt{x^2+a^2}}dx = \sqrt{x^2+a^2} + C}$ **Ex** $\int \frac{x}{\sqrt{x^2+25}}dx = \sqrt{x^2+5^2} + C$

19. $\boxed{\int \frac{x}{\sqrt{x^2-a^2}}dx = \sqrt{x^2-a^2} + C}$ **Ex** $\int \frac{x}{\sqrt{x^2-36}}dx = \sqrt{x^2-36} + C$

20. $\boxed{\int \frac{x}{\sqrt{a^2-x^2}}dx = -\sqrt{a^2-x^2}+C}$ **Ex** $\int \frac{x}{\sqrt{25-x^2}}dx = -\sqrt{25-x^2}+C$

21. $\boxed{\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\ln\left|x+\sqrt{x^2+a^2}\right|+C}$ **Ex** $\int \sqrt{x^2+7}\,dx = \frac{x}{2}\sqrt{x^2+7}+\frac{7}{2}\ln\left|x+\sqrt{x^2+7}\right|+C$

22. $\boxed{\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right|+C}$ **Ex** $\int \sqrt{x^2-9}\,dx = \frac{x}{2}\sqrt{x^2-9}-\frac{9}{2}\ln\left|x+\sqrt{x^2-9}\right|+C$

23. $\boxed{\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\arcsin\frac{x}{a}+C}$ **Ex** $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\arcsin\frac{x}{a}+C$

I. Să se calculeze primitivele următoarelor funcţii.

1. $\int (3x^5-2x^3+3x-2)dx$

2. $\int x(x-1)(x-2)dx$

3. $\int (\sqrt{x}+1)(x-\sqrt{x}+1)dx$

4. $\int (\sqrt[3]{x}+\frac{1}{x\sqrt[3]{x}})dx$

5. $\int \left(2\sqrt{x}-\sqrt[3]{x}+4\sqrt[5]{x}\right)dx$

6. $\int \left(\frac{5}{\sqrt[5]{x}}-\frac{3}{\sqrt[3]{x}}+\frac{2}{\sqrt{x}}\right)dx$

7. $\int x\sqrt{(x-1)^3}\,dx$

8. $\int \left(2x+\frac{5}{x}-\frac{3}{x^2}\right)dx$

9. $\int (e^x+\frac{1}{e^x})dx$

10. $\int (x^5+5^x)dx$

11. $\int \left(\frac{5+4x}{x}\right)^2 dx$

12. $\int \frac{(x+2)^3}{x^3}dx$

13. $\int \sqrt{x^2+4}\,dx$

14. $\int \sqrt{x^2-9}\,dx$

15. $\int \sqrt{4-x^2}\,dx$

16*. $\int \frac{1}{x+\sqrt{x^2-1}}dx$

17*. $\int \frac{x^2+3}{\sqrt{x^2+2}}dx$

18*. $\int \frac{x^2-2}{\sqrt{x^2-3}}dx$

19*. $\int \frac{1}{\sin^2 x\cdot\cos^2 x}dx$

20*. $\int \frac{1}{\sin x\cdot\cos x}dx$

21*. $\int \sqrt{\frac{1+x}{1-x}}dx$

3. PRIMITIVELE FUNCŢIILOR CONTINUE COMPUSE

1. $\boxed{\int \varphi'(x)dx = \varphi(x) + C, \quad \varphi(x) \in R}$ **Ex** $\int (5x+1)' dx = 5x+1+C$

2. $\boxed{\int \varphi(x)\cdot\varphi'(x)dx = \frac{\varphi(x)^2}{2} + C}$ **Ex** $\int (4x+3)\cdot 4dx = \frac{(4x+3)^2}{2} + C$

3. $\boxed{\int \varphi(x)^n\cdot\varphi'(x)dx = \frac{\varphi(x)^{n+1}}{n+1} + C}$ **Ex** $\int (5x+2)^7\cdot 5dx = \frac{(5x+2)^8}{8} + C$

4. $\boxed{\int e^{\varphi(x)}\cdot\varphi'(x)dx = e^{\varphi(x)} + C}$ **Ex** $\int e^{2x+4}\cdot 2dx = e^{2x+4} + C$

5. $\boxed{\int a^{\varphi(x)}\cdot\varphi'(x)dx = \frac{a^{\varphi(x)}}{\ln a} + C}$ **Ex** $\int 4^{3x}\cdot 3dx = \frac{4^{3x}}{\ln 4} + C$

6. $\boxed{\int \frac{\varphi'(x)}{\varphi(x)}dx = \ln|\varphi(x)| + C}$ **Ex** $\int \frac{12}{12x+7}dx = \ln|12x+7| + C$

7. $\boxed{\int \frac{\varphi'(x)}{\varphi^n(x)}dx = \frac{-1}{n-1}\cdot\frac{1}{\varphi^{n-1}(x)} + C}$ **Ex** $\int \frac{2}{(2x-4)^6}dx = \frac{-1}{5}\cdot\frac{1}{(2x-4)^5} + C$

8. $\boxed{\int \frac{\varphi'(x)}{\varphi^2(x)-a^2}dx = \frac{1}{2a}\ln\left|\frac{\varphi(x)-a}{\varphi(x)+a}\right| + C}$ **Ex** $\int \frac{4}{16x^2-9}dx = \frac{1}{6}\ln\left|\frac{4x-3}{4x+3}\right| + C$

9. $\boxed{\int \frac{\varphi'(x)}{\varphi^2(x)+a^2}dx = \frac{1}{a}arctg\frac{\varphi(x)}{a} + C}$ **Ex** $\int \frac{5}{25x^2+4}dx = \frac{1}{2}arctg\frac{5x}{2} + C$

10. $\boxed{\int \varphi'(x)\cdot\sin\varphi(x)dx = -\cos\varphi(x) + C}$ **Ex** $\int 4\cdot\sin(4x-5)dx = -\cos(4x-5) + C$

11. $\boxed{\int \varphi'(x)\cdot\cos\varphi(x)dx = \sin\varphi(x) + C}$ **Ex** $\int 6x\cdot\cos(3x^2+7)dx = \sin(3x^2+7) + C$

12. $\boxed{\int \varphi'(x)\cdot tg\varphi(x)dx = -\ln|\cos\varphi(x)| + C}$ **Ex** $\int 5\cdot tg(5x-7)dx = -\ln|\cos(5x-7)| + C$

13. $\boxed{\int \varphi'(x)\cdot ctg\varphi(x)dx = \ln|\sin\varphi(x)| + C}$ **Ex** $\int 8\cdot ctg(8x+6)dx = \ln|\sin(8x+6)| + C$

14. $\boxed{\int \frac{\varphi'(x)}{\cos^2\varphi(x)}dx = tg\,\varphi(x) + C}$ **Ex** $\int \frac{6}{\cos^2 6x}dx = tg6x + C$

15. $\boxed{\int \frac{\varphi'(x)}{\sin^2\varphi(x)}dx = -ctg\,\varphi(x) + C}$ **Ex** $\int \frac{5}{\sin^2(5x-9)}dx = -ctg(5x-9) + C$

16. $\boxed{\int \frac{\varphi(x)\varphi'(x)}{\sqrt{\varphi^2(x)-a^2}}dx = \sqrt{\varphi^2(x)-a^2} + C}$ **Ex** $\int \frac{3x\cdot 3}{\sqrt{9x^2-4}}dx = \sqrt{9x^2-4} + C$

17. $\int \frac{\varphi(x)\varphi'(x)}{\sqrt{\varphi^2(x)+a^2}}dx = \sqrt{\varphi^2(x)+a^2}+C$ **Ex** $\int \frac{4x\cdot 4}{\sqrt{16x^2+25}}dx = \sqrt{16x^2+25}+C$

18. $\int \frac{\varphi(x)\varphi'(x)}{\sqrt{a^2-\varphi^2(x)}}dx = -\sqrt{a^2-\varphi^2(x)}+C$ **Ex** $\int \frac{2x\cdot 2}{\sqrt{9-4x^2}}dx = -\sqrt{9-4x^2}+C$

19. $\int \frac{\varphi'(x)}{\sqrt{\varphi(x)^2+a^2}}dx = \ln(\varphi(x)+\sqrt{\varphi(x)^2+a^2})+C$ **Ex** $\int \frac{5}{\sqrt{25x^2+7^2}}dx = \ln(5x+\sqrt{25x^2+7^2})+C$

20. $\int \frac{\varphi'(x)}{\sqrt{\varphi(x)^2-a^2}}dx = \ln\left|\varphi(x)+\sqrt{\varphi(x)^2-a^2}\right|+C$ **Ex** $\int \frac{3}{\sqrt{9x^2-4^2}}dx = \ln\left|3x+\sqrt{9x^2-4^2}\right|+C$

21. $\int \frac{\varphi'(x)}{\sqrt{a^2-\varphi(x)^2}}dx = \arcsin\frac{\varphi(x)}{a}+C$ **Ex** $\int \frac{2}{\sqrt{5^2-4x^2}}dx = \arcsin\frac{2x}{5}+C$

22. $\int \sqrt{\varphi(x)^2+a^2}\,dx = \frac{\varphi(x)}{2}\sqrt{\varphi(x)^2+a^2}+\frac{a^2}{2}\ln\left|\varphi(x)+\sqrt{\varphi(x)^2+a^2}\right|+C$

23. $\int \sqrt{\varphi(x)^2-a^2}\,dx = \frac{\varphi(x)}{2}\sqrt{\varphi(x)^2-a^2}-\frac{a^2}{2}\ln\left|\varphi(x)+\sqrt{\varphi(x)^2-a^2}\right|+C$

24. $\int \sqrt{a^2-\varphi(x)^2}\,dx = \frac{\varphi(x)}{2}\sqrt{a^2-\varphi(x)^2}+\frac{a^2}{2}\arcsin\frac{\varphi(x)}{a}+C$

II.Să se calculeze primitivele următoarelor funcţii compuse.

1. $\int 5\cdot 2^{5x}dx$ **2.** $\int 3^{4x}dx$ **3.** $\int 4\sin 4x dx$ **4.** $\int 3\cos 3x dx$

5. $\int \frac{1}{5x+3}dx$ **6.** $\int \frac{1}{4x^2+9}dx$ **7.** $\int \frac{1}{4x^2-16}dx$ **8.** $\int \frac{1}{25-9x^2}dx$

9. $\int \frac{1}{\cos^2 3x}dx$ **10.** $\int \frac{1}{\sin^2 5x}dx$ **11.** $\int tg4x dx$ **12.** $\int 2ctg2x dx$

13. $\int \frac{1}{\sqrt{16x^2+4^2}}dx$ **14.** $\int \frac{1}{\sqrt{9-16x^2}}dx$

III. Să se arate că următoarele funcţii nu admit primitive.

1. f: R → R, f(x) = $\begin{cases} -1, & x < 0 \\ 1, & x \geq 0 \end{cases}$
2. f: R → R , f(x) = [x] (partea întreagă din x)

3. f: R → R, f(x) = $\begin{cases} -1, x\langle 0 \\ 0, x = 0 \\ 1, x\rangle 0 \end{cases}$

4. f: R → R , f(x) = [X] +X

5. . f: R → R f(x) = $\begin{cases} -1, x \in (-\infty,0] \\ x+1, x \in (0,+\infty) \end{cases}$

6. f: R → R , f(x) = $\begin{cases} \sin x, x \neq 0 \\ 2, x = 0 \end{cases}$

IV. Să se determine a,b numere reale astfel încât F să fie primitiva unei funcţii f.

1*. F(x) = $\begin{cases} \ln(ax+b), x\rangle 1 \\ \dfrac{x+1}{x^2+1}, x \leq 1 \end{cases}$

2*. F(x) = $\begin{cases} 1+\ln^2 x, x \in [1,e) \\ (2a-3)x+b^2, x \in [e,e^2] \end{cases}$

3*. F(x) = $\begin{cases} 2a\cdot e^{3x}+b, x \leq 0 \\ \sqrt{2x^2-4x+1}, x\rangle 0 \end{cases}$

4*. F(x) = $\begin{cases} \dfrac{ax+b}{x^2+1}, x \leq 0 \\ \sqrt{x^2-6x+9}, x\rangle 0 \end{cases}$

5*. F(x) = $\begin{cases} a\cdot e^{2x}+2bx+1, x \leq 0 \\ \dfrac{3x+a}{x^2+2}, x\rangle 0 \end{cases}$

6*. F(x) = $\begin{cases} x^2+ax+b, x\rangle 0 \\ \sin x+3\cos x, x \leq 0 \end{cases}$

V. Să se verifice dacă următoarele funcţii admit primitive şi în caz afirmativ să se determine o primitivă.

1. f: R→ R, f(x) = $\begin{cases} x^2+3, x\langle 0 \\ e^x+2, x \geq 0 \end{cases}$

2. f: R→ R, f (x) = $\begin{cases} \dfrac{1}{x^2+4}, x \leq 0 \\ \dfrac{1}{4}-\sqrt{x}, x\rangle 0 \end{cases}$

3*.f:[0,∞)→R, f(x) = $\begin{cases} \sqrt{x}+\sqrt[3]{x}, x \in [0,1) \\ \dfrac{1}{\sqrt{x}}+\dfrac{1}{\sqrt[3]{x}}, x \geq 1 \end{cases}$

4*. f:[-2,∞)→R, f(x) = $\begin{cases} \dfrac{x^3+2^x}{3}, x \geq 0 \\ \dfrac{1}{\sqrt{9-x^2}}, x \in [-2,0) \end{cases}$

DERIVATE

Nr	FUNCTIA	DERIVATA	MULTIMEA PE CARE FUNCTIA ESTE DERIVABILĂ	FUNCTIA COMPUSĂ	DERIVATA
1.	**C**	0	R		
2.	**x**	1	R	u	u'
3.	$\mathbf{x^n}$	nx^{n-1}	R	u^n	$n.u^{n-1}.u'$
4.	$\mathbf{x^a}$	ax^{a-1}	$[0,\infty]$	u^a	$au^{a-1}.u'$
5.	$\frac{1}{x}$	$-\frac{1}{x^2}$	R^*	$\frac{1}{u}$	$-\frac{1}{u^2}.u'$
6.	$\frac{1}{x^n}$	$-\frac{n}{x^{n+1}}$	R^*	$\frac{1}{u^n}$	$-n/u^{n+1}\cdot u'$
7.	$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$	R^*_+	$\sqrt{u}$	$\frac{1}{2\sqrt{u}}u'$
8.	$\sqrt[n]{x}$	$\frac{1}{n\sqrt[n]{x^{n-1}}}$	$R^*_{+,n\ par}$ $R^*_{,n\ impar}$	$\sqrt[n]{u}$	$\frac{1}{n\sqrt[n]{u^{n-1}}}u'$
9.	**sin x**	cosx	R	sin u	u'cos u
10.	**cos x**	-sinx	R	cos u	-u'sin u
11.	**tg x**	$\frac{1}{\cos^2 x}$	$R\backslash\{(2k+1)\frac{\pi}{2} \mid k\in Z\}$	tg u	$\frac{1}{\cos^2 u}u'$
12.	**ctg x**	$-\frac{1}{\sin^2 x}$	$R\backslash\{k\pi \mid k\in Z\}$	ctg u	$-\frac{1}{\sin^2 u}u'$
13.	**arcsin x**	$\frac{1}{\sqrt{1-x^2}}$	(-1,1)	arcsin u	$\frac{1}{\sqrt{1-u^2}}u'$
14.	**arccos x**	$-\frac{1}{\sqrt{1-x^2}}$	(-1,1)	arccos u	$-\frac{1}{\sqrt{1-u^2}}u'$
15.	**arctg x**	$\frac{1}{1+x^2}$	R	arctg u	$\frac{1}{1+u^2}u'$
16.	**arcctg x**	$-\frac{1}{1+x^2}$	R	arcctg u	$-\frac{1}{1+u^2}u'$
17.	$\mathbf{a^x}$	$a^x \ln a$	R	a^u	$a^u.\ln a.u'$
18.	$\mathbf{e^x}$	e^x	R	e^u	$e^u.u'$
19.	**lnx**	$\frac{1}{x}$	R^*_+	lnu	$\frac{1}{u}.u'$
20.	$\mathbf{\log_a x}$	$\frac{1}{x\ln a}$	R^*_+	$\log_a u$	$\frac{1}{u\ln a}u'$
21.	$\mathbf{u^v}$	$(u^v)' =$	$v.u^{v-1}.u' + u^v.v'.\ln u$		

METODE DE CALCUL AL INTEGRALELOR

1. Formula de integrare prin părţi.

Teorema 1.1 Dacă f,g:R→R sunt funcţii derivabile cu derivatele continue, atunci funcţiile fg, f'g, fg' admit primitive şi are loc relaţia: $\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$

Demonstraţie: f,g derivabile $\Rightarrow$ f,g continue $\Rightarrow$ f'g,fg,fg' continue şi deci admit primitive.
Cum **(fg)'=f'g+g'f** rezultă prin integrare ceea ce trebuia de demonstrat.

Să se calculeze integralele:

1. $\int \ln x dx$ 2. $\int x \ln x dx$ 3. $\int x^2 \cdot \ln x dx$ 4. $\int \frac{1}{x} \ln x dx$

5. $\int \frac{1}{x^2} \ln x dx$ 6. $\int \frac{\ln(\ln x)}{x} dx$ 7. $\int \ln^2 x dx$ 8. $\int \ln(1+\frac{2}{x}) dx$

9*. $\int \frac{\ln^3 x}{x^2} dx$ 10. $\int \frac{\ln^2 x}{x^2} dx$ 11. $\int \cos(\ln x) dx$ 12. $\int \sin(\ln x) dx$

13. $\int (x^2 - 2x + 3) \ln x dx$ 14. $\int x \ln(x-1) dx$ 15. $\int \frac{x}{\sqrt{x^2+1}} \ln(1+\sqrt{x^2+1}) dx$

16*. $\int x \ln \frac{x-1}{x+1} dx$ 17. $\int (x^2+1) \cdot e^x dx$ 18. $\int x \cdot e^{-x} dx$

19. $\int (x^2 + 2x) \cdot e^{3x} dx$ 20. $\int x^2 \cdot e^x dx$ 21. $\int x^2 \cdot e^{2x} dx$

22*. $\int (x^3 + 5x^2 - 2) \cdot e^{2x} dx$ 23. $\int x^2 \cdot e^{-x} dx$ 24*. $\int \frac{3 \cdot 2^x + 2 \cdot e^x}{2^x} dx$

25. $\int e^x \cdot \sin x dx$ 26. $\int e^x \cdot \cos x dx$ 27. $\int e^x \cdot \sin 2x dx$

28. $\int e^x \cdot \cos 2x dx$ 29. $\int x \cdot \sin x dx$ 30. $\int x \cdot \cos x dx$

31. $\int x^2 \cdot \sin x dx$ 32. $\int x^2 \cdot \cos x dx$ 33*. $\int x^2 \cdot \sin 2x dx$

34*. $\int x^2 \cdot \cos 2x dx$ 35. $\int x \cdot \sin^2 x dx$ 36. $\int x \cdot \cos^2 x dx$

37. $\int \frac{x}{\cos^2 x} dx$ 38. $\int \frac{x}{\sin^2 x} dx$ 39. $\int \frac{x \cdot \arcsin x}{\sqrt{1-x^2}} dx$

40. $\int \frac{\arcsin x}{x^2} dx$ 41*. $\int e^{-x} \cdot \sin^2 x dx$ 42*. $\int \cos^2(\ln x) dx$

43*. $\int x \cdot \sqrt{x^2 - 9} dx$ 44*. $\int x \cdot \sqrt{x^2 + 16} dx$ 45*. $\int x \cdot \sqrt{4 - x^2} dx$

46. $\int \sqrt{x} \ln x dx$ 47*. $\int \frac{x^2 - 2x + 5}{e^x} dx$

Rezolvări:

1. $\int \ln x dx = \int x' \ln x dx = x \cdot \ln x - \int x \cdot \frac{1}{x} dx = x \ln x - \int dx = x \ln x - x + C$

2. $\int x \cdot \ln x dx = \int \left(\frac{x^2}{2} \right)' \ln x dx = \frac{x^2}{2} \cdot \ln x - \int \frac{x^2}{2} \cdot \frac{1}{x} dx = \frac{x^2}{2} \ln x - \frac{1}{2} \int x dx = \frac{x^2}{2} \ln x - \frac{1}{4} x^2 + C$

4.
$\int \frac{1}{x} \ln x dx = \int (\ln x)' \ln x dx = (\ln x)^2 - \int \ln x \cdot \frac{1}{x} dx \Rightarrow 2 \int \ln x \cdot \frac{1}{x} dx = (\ln x)^2 \Rightarrow \int \frac{1}{x} \ln x dx = \frac{1}{2} (\ln x)^2 + C$

Observaţie: La integralele care conţin funcţia logaritmică nu se umblă la ea ci se scriu celelalte funcţii ca f ’

20. $\int x^2 \cdot e^x dx = \int x^2 (e^x)' dx = x^2 e^x - \int 2x \cdot e^x dx = x^2 e^x - 2 \int x \cdot (e^x)' dx = x^2 e^x - 2[x \cdot e^x - \int e^x dx] =$
$= x^2 e^x - 2x \cdot e^x + 2e^x + C = e^x (x^2 - 2x + 2) + C$

25. $\int e^x \cdot \sin x dx = \int (e^x)' \sin x dx = e^x \sin x - \int e^x \cdot \cos x dx = e^x \sin x - [e^x \cdot \cos x - \int e^x \cdot (-\sin x) dx] \Rightarrow$

Notând cu I integrala $\int e^x \cdot \sin x dx$ rezultă: $I = e^x \sin x - e^x \cos x - I \Rightarrow I = \frac{1}{2} e^x (\sin x - \cos x) + C$

Observaţie: La integralele unde apare funcţia exponenţială , se va scrie aceasta ca f ’

29. $\int x \cdot \sin x dx = \int x \cdot (-\cos x)' dx = -x \cos x + \int \cos x dx = -x \cos x + \sin x + C$

32. $\int x^2 \cdot \cos x dx = \int x^2 \cdot (\sin x)' dx = x^2 \sin x - \int 2x \cdot \sin x dx = x^2 \sin x - 2(-x \cos x + \sin x) + C, vezi\ \ 29$

37. $\int \frac{x}{\cos^2 x} dx = \int (tgx)' \cdot x dx = x \cdot tgx - \int tgx dx = x \cdot tgx - (-\ln|\cos x|) = x \cdot tgx + \ln|\cos x| + C$

Observaţie: La integralele care conţin funcţii polinomiale şi funcţii trigonometrice nu se va umbla la funcţiile polinomiale ci doar la funcţiile trigonometrice care se vor scrie ca f ’

41*. Se ştie că: $\cos 2x = 1 - 2\sin^2 x \Rightarrow \sin^2 x = \dfrac{1-\cos 2x}{2} \Rightarrow$

$$\int e^{-x}\cdot \sin^2 x dx = \int e^{-x}\cdot\left(\frac{1-\cos 2x}{2}\right)dx = \frac{1}{2}\int e^{-x}dx - \frac{1}{2}\int e^{-x}\cos 2x dx = \frac{-e^{-x}}{2} - \frac{1}{2}I$$

$$I = \int e^{-x}\cos 2x dx = \int e^{-x}\left(\frac{\sin 2x}{2}\right)' dx = \frac{1}{2}e^{-x}\sin 2x - \int\left(-e^{-x}\right)\frac{\sin 2x}{2}dx = \frac{1}{2}e^{-x}\sin 2x + \frac{1}{2}\int e^{-x}\sin 2x dx =$$

$$= \frac{1}{2}e^{-x}\sin 2x + \frac{1}{2}\int e^{-x}\left(\frac{-\cos 2x}{2}\right)' dx = \frac{1}{2}e^{-x}\sin 2x - \frac{e^{-x}\cos 2x}{4} + \frac{1}{4}\int e^{-x}\cos 2x dx$$

$$\Rightarrow I = \frac{1}{2}e^{-x}\sin 2x - \frac{e^{-x}\cos 2x}{4} + \frac{1}{4}I \Rightarrow I = \frac{4}{3}\left(\frac{1}{2}e^{-x}\sin 2x - \frac{e^{-x}\cos 2x}{4}\right) + C$$

METODE DE CALCUL AL INTEGRALELOR

2. FORMULA SCHIMBĂRII DE VARIABILĂ (SAU METODA SUBSTITUŢIEI).

Teoremă: Fie I,J intervale din R şi $\varphi : I \to J, f : J \to R$, *functii cu proprietatile* :

1) φ este derivabilă pe I;

2) f admite primitive. (Fie F o primitivă a sa.)

Atunci funcţia $(f \circ \varphi)\varphi'$ admite primitive, iar funcţia $F \circ \varphi$ este o primitivă a lui $(f \circ \varphi)\varphi'$ adică:

$$\boxed{\int f(\varphi(t))\cdot\varphi'(t)dt = F o\varphi + C}$$

Să se calculeze integralele:

1. $\int (ax+b)^n dx$
2. $\int (2x-1)^9 dx$
3. $\int x(2x-1)^9 dx$
4. $\int x(5x^2-3)^7 dx$
5. $\int x^2(x^3+1)^6 dx$
6. $\int x^k(x^{k+1}+1)^n dx$
7. $\int x\cdot 7^{x^2} dx$
8. $\int \frac{e^x}{e^x+1}dx$
9. $\int \frac{e^x}{e^{2x}+1}dx$
10. $\int e^{\sqrt{x}}dx$
11. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$
12. $\int \frac{e^{2x}}{e^x-1}dx$
13. $\int \frac{e^{3x}}{e^{2x}-1}dx$
14. $\int x\sqrt{x-1}dx$
15. $\int \sqrt{2x+5}dx$
16. $\int x\sqrt{1+x^2}dx$
17. $\int x^3\sqrt{1-x^4}dx$
18. $\int x^2\sqrt[5]{x^3+2}dx$
19. $\sqrt[3]{2x+5}dx$
20. $\int \sqrt{x^2-6x-7}dx$
21. $\int \sqrt{-x^2-x+2}dx$
22. $\int \frac{\sqrt{\ln x}}{x}dx$
23. $\int \frac{\ln\sqrt{x}}{x}dx$
24. $\int \sqrt{x}\ln x dx$
25. $\int \frac{x-2\sqrt{x}}{\sqrt[3]{x}}dx$
26. $\int \frac{(1-x)^2}{x\sqrt{x}}dx$
27. $\int \frac{1}{\sqrt{4x^2+2x-3}}dx$

28. $\int \frac{1}{\sqrt{-x^2+3x+4}}dx$ 29. $\int \frac{x}{\sqrt[4]{x+1}}dx$ 30. $\int \frac{x}{\sqrt{x^2+1}}dx$ 31.
$\int \frac{1}{x(1+\ln x)^4}dx$ 32. $\int \frac{1}{x(\ln^2 x+8)}dx$ 33. $\int \frac{1}{x\sqrt{3-\ln^2 x}}dx$ 34.
$\int \frac{1}{x\ln x}dx$ 35. $\int \frac{\sqrt[3]{1+\ln x}}{x}dx$ 36 . $\int x^3\sqrt{x^2+2}dx$ 37.
$\int \frac{1}{x(2005+\ln x)^{2006}}dx$ 38. $\int \frac{1}{x\sqrt{x^2-1}}dx$

Rezolvări:

1. $\int (ax+b)^n dx = \int t^n \frac{dt}{a} = \frac{1}{a}\frac{t^{n+1}}{n+1} + C = \frac{(ax+b)^{n+1}}{a(n+1)} + C$ unde ax+b=t $\Rightarrow$adx=dt dx=$\frac{dt}{a}$

2. $\int (2x-1)^9 dx = \int t^9 \frac{dt}{2} = \frac{t^{10}}{20} + C = \frac{(2x-1)^{10}}{20} + C$ unde 2x-1=t $\Rightarrow$2dx=dt

3. $\int x(2x-1)^9 dx = \int \frac{t+1}{2}\cdot t^9 dt = \frac{1}{2}\int (t^{10}+t^9)dt = \frac{t^{11}}{22} + \frac{t^{10}}{20} + C = \frac{(2x-1)^{11}}{22} + \frac{(2x-1)^{10}}{20} + C$

4. $\int x(5x^2-3)^7 dx = \int t^7 \cdot \frac{dt}{10} = \frac{1}{10}\cdot\frac{t^8}{8} + C = \frac{(5x^2-3)^8}{80} + C, unde 5x^2-3=t \Rightarrow 10xdx = dt$

7. $\int x\cdot 7^{x^2} dx = \int 7^t \frac{dt}{2} = \frac{1}{2}\frac{7^t}{\ln 7} + C = \frac{1}{2}\frac{7^{x^2}}{\ln 7} + C, unde\ \ x^2 = t,\ \ 2xdx = dt$

8. $\int \frac{e^x}{e^x+1}dx$ Notăm: $e^x+1 = t \Rightarrow e^x dx = dt \Rightarrow$ $\int \frac{e^x}{e^x+1}dx = \int \frac{1}{t}dt = \ln t + C = \ln(e^x+1) + C$

15. $\int \sqrt{2x+5}dx$ Notăm: $\sqrt{2x+5} = t\ \ sau\ \ 2x+5 = t^2 \Rightarrow 2dx = 2tdt \Rightarrow dx = tdt$

$$\int \sqrt{2x+5}dx = \int t\cdot tdt = \frac{t^3}{3} + C = \frac{\sqrt{2x+5}^3}{3} + C$$

20. $\int \sqrt{x^2-6x-7}dx = \sqrt{(x-3)^2 - \left(\frac{8}{2}\right)^2} = \frac{x-3}{2}\sqrt{x^2-6x-7} - \frac{16}{2}\ln\left|x-3+\sqrt{x^2-6x-7}\right| + C$

deoarece:

$$\boxed{\begin{array}{l} ax^2+bx+c = a\left[\left(x+\frac{b}{2a}\right)^2 - \left(\frac{\sqrt{\Delta}}{2a}\right)^2\right] daca\ \ \Delta\rangle 0\ \ sau \\ ax^2+bx+c = a\left[\left(x+\frac{b}{2a}\right)^2 + \left(\frac{\sqrt{-\Delta}}{2a}\right)^2\right] daca\ \ \Delta\langle 0 \end{array}}$$

23. $\int \frac{\ln\sqrt{x}}{x}dx = \int \frac{\ln t}{t^2}\cdot 2tdt = 2\int \frac{\ln t}{t}dt = 2\int zdz$ *deoarece* $x = t^2 \Rightarrow dx = 2tdt, \ln t = z \Rightarrow \frac{1}{t}dt = dz \Rightarrow$

$$\int \frac{\ln\sqrt{x}}{x}dx = 2\frac{z^2}{2} = (\ln t)^2 = (\ln\sqrt{x})^2 + C$$

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$$\int \frac{1}{\sqrt{-x^2+3x+4}}dx = \int \frac{1}{\sqrt{\left(\frac{5}{2}\right)^2 - \left(x-\frac{3}{2}\right)^2}}dx = \arcsin\frac{\frac{2x-3}{2}}{\frac{5}{2}} + C = \arcsin\frac{2x-3}{5} + C, \textit{puteamnota } \frac{2x-3}{2} \textit{ cu } t$$

INTEGRAREA FUNCŢIILOR TRIGONOMETRICE

Calculul integralelor trigonometrice se poate face fie folosind **formula integrării prin părţi**, fie **metoda substituţiei**. În acest caz se pot face substituţiile:
1. Dacă funcţia este impară în sin x, R(-sin x,cos x)=-R(sin x,cos x) atunci cos x=t.
2. Dacă funcţia este impară în cos x, R(sin x,-cos x)=-R(sin x,cos x) atunci sin x=t.
3. Dacă funcţia este pară în raport cu ambele variabile R(-sin x,-cos x) atunci tg x=t.
4. Dacă o funcţie nu se încadrează în cazurile 1,2,3,atunci se utilizează substituţiile universale:

$$\sin x = \frac{2t}{1+t^2}, \cos x = \frac{1-t^2}{1+t^2} \quad \textit{unde } t = tg\frac{x}{2}$$

5. Se mai pot folosi şi alte formule trigonometrice:

sin 2x=2sin x .cos x, $\sin^2 x = \frac{1-\cos 2x}{2}$ $\quad \cos^2 x = \frac{1+\cos 2x}{2}$

Să se calculeze:

1. $\int \sin^3 x\cdot\cos xdx$ 2. $\int \cos^3 x\cdot\sin 2xdx$ 3. $\int \sin(2x+5)dx$

4. $\int \sin^3 x\cdot\cos^2 xdx$ 5. $\int (tgx + tg^3 x)dx$ 6. $\int \frac{\cos x}{1+\sin^2 x}dx$

7. $\int \frac{\sin^3 x}{\cos x}dx$ 8. $\int \frac{1}{\sqrt{x}}\cos\sqrt{x}dx$ 9. $\int \frac{x}{1-\cos x}dx$

10. $\int \sin^3 xdx$ 11. $\int \cos^3 xdx$ 12. $\int \frac{\arcsin x}{\sqrt{1-x^2}}dx$

13. $\int \frac{\sin x}{\cos^2 x - 4}dx$ 14. $\int \frac{\sin 2x}{\sqrt{1-(\cos^2 x)^2}}dx$ 15. $\int \frac{1}{\sqrt{1-x^2}\cdot\arcsin^2 x}dx$

16. $\int \frac{1}{\sin x}dx$ 17. $\int \frac{1}{\cos x}dx$ 18. $\int \sin^{10} x\cdot\cos^3 xdx$

19. $\int \frac{1}{\sqrt{1-x^2}\cdot(2005+\arcsin x)^{2006}}dx$ 20. $\int \frac{(arctgx)^{2006}}{1+x^2}dx$

Rezolvări:

1. Notăm sin x=t $\Rightarrow$ cosx dx= dt $\Rightarrow \int \sin^3 x\cdot\cos x dx=\int t^3 dt=\frac{t^4}{4}+C=\frac{\cos^4 x}{4}+C$
2. Notăm cos x=t $\Rightarrow$-sin x dx=dt

$$\int \cos^3 x\cdot\sin 2x dx=\int\cos^3 x\cdot 2\sin x\cos x dx=2\int\cos^4 x\sin x dx=-2\int t^4 dt=-2\cdot\frac{t^5}{5}+C=-\frac{2}{5}\cdot\cos^5 x+C$$

10. $\int \sin^3 x dx=\int \sin x\sin^2 x dx=\int \sin x(1-\cos^2 x)dx=-\int(1-t^2)dt=-t+\frac{t^3}{3}+C=-\cos x+\frac{\cos^3 x}{3}+C$

12. $\int \frac{\arcsin x}{\sqrt{1-x^2}}dx$ Notăm cu t pe arcsin x $\Rightarrow (\arcsin x)'=\frac{1}{\sqrt{1-x^2}}\Rightarrow \frac{1}{\sqrt{1-x^2}}dx=dt\Rightarrow$

$$\int \frac{\arcsin x}{\sqrt{1-x^2}}dx=\int t dt=\frac{t^2}{2}+C=\frac{(\arcsin x)^2}{2}+C$$

INTEGRAREA FUNCŢIILOR RAŢIONALE

Definiţie: O funcţie f:I→R , I interval, se numeşte raţională dacă R(x)= $\frac{f(x)}{g(x)}, g(x)\neq 0, x\in I$, **unde f,g sunt funcţii polinomiale.**

Dacă grad f $\geq$grad g, atunci se efectuează împărţirea lui f la g $\Rightarrow$f=gq+r, 0$\leq$grad r<grad g şi deci

$R(x)=\frac{f(x)}{g(x)}=q(x)+\frac{r(x)}{g(x)}$. *Pentru* $R(x)$ *se face scrierea ca sumă de functii rationale simple.*

1. $\boxed{\int\frac{1}{ax+b}dx=\frac{1}{a}\ln|ax+b|+C}$ **Ex.** $\int\frac{1}{2x-7}dx=\frac{1}{2}\ln|2x-7|+C$

2. $\boxed{\int\frac{1}{(ax+b)^n}dx=-\frac{1}{(n-1)(ax+b)^{n-1}}\cdot\frac{1}{a}+C}$ **Ex** $\int\frac{1}{(3x+8)^7}dx=-\frac{1}{6(3x+8)^6}\cdot\frac{1}{3}+C$

3. $\boxed{\int\frac{1}{x^2+a^2}dx=\frac{1}{a}arctg\frac{x}{a}+C}$ **Ex** $\int\frac{1}{x^2+5^2}dx=\frac{1}{5}arctg\frac{x}{5}+C$

4. $\boxed{\int\frac{1}{x^2-a^2}dx=\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+C}$ **Ex** $\int\frac{1}{x^2-25}dx=\frac{1}{10}\ln\left|\frac{x-5}{x+5}\right|+C$

5* $\boxed{\int\frac{1}{(x^2+a^2)^2}dx=\frac{1}{a^2}\int\frac{x^2+a^2-x^2}{(x^2+a^2)^2}+C=\frac{1}{a^2}\int\frac{1}{x^2+a^2}dx-\frac{1}{a^2}\int x\cdot\left(\frac{-1}{2(x^2+a^2)}\right)'dx}$

Ex. $$\int \frac{1}{(x^2+16)^2}dx = \int \frac{x^2+16-x^2}{\left(x^2+16\right)^2} + C = \frac{1}{16}\int \frac{1}{x^2+16}dx - \frac{1}{16}\int x\cdot\left(\frac{-1}{2\left(x^2+16\right)}\right)'dx$$

$$\frac{1}{16}\cdot\frac{1}{4}arctg\ \frac{x}{4} - \frac{1}{16}[-\frac{x}{2x^2+32} + \int \frac{1}{2(x^2+4^2)}]$$

6. $$\boxed{\int \frac{1}{ax^2+bx+c}dx = \begin{cases} \int \dfrac{1}{a[(x+\frac{b}{2a})^2-(\frac{\sqrt{\Delta}}{2a})^2]}dx, & \Delta\rangle 0 \\ \int \dfrac{1}{a[(x+\frac{b}{2a})^2+(\frac{\sqrt{-\Delta}}{2a})^2]}dx, & \Delta\langle 0 \end{cases}}$$

Ex. $$\int \frac{1}{4x^2-5x+1}dx = \int \frac{1}{4\left[\left(x-\frac{5}{8}\right)^2-\left(\frac{3}{8}\right)^2\right]}dx = \frac{1}{4}\cdot\frac{1}{2\cdot\frac{3}{8}}\ln\left|\frac{x-\frac{5}{8}-\frac{3}{8}}{x-\frac{5}{8}+\frac{3}{8}}\right| + C = \frac{1}{3}\ln\left|\frac{8x-8}{8x-2}\right| + C$$

Ex. $$\int \frac{1}{x^2+4x+5}dx = \int \frac{1}{(x+2)^2+1}dx = arctg(x+1)+C$$

7. $$\boxed{\int \frac{2ax+b}{ax^2+bx+c}dx = \ln\left|ax^2+bx+c\right| + C}$$

Ex. $$\int \frac{8x-6}{4x^2-6x+7}dx = \ln\left|4x^2-6x+7\right| + C$$

8*. $$\boxed{\int \frac{Ax+B}{ax^2+bx+c}dx = \int \frac{m(2ax+b)+n}{ax^2+bx+c}dx = m\cdot\ln\left|ax^2+bx+c\right| + n\cdot\int \frac{1}{ax^2+bx+c}dx}$$

Ex.

$$\int \frac{3x+4}{2x^2+5x-4}dx = \int \frac{\frac{3}{4}\cdot(4x+5)+4-\frac{15}{4}}{2x^2+5x-4}dx = \frac{3}{4}\ln\left|2x^2+5x-4\right| + \frac{1}{4}\int \frac{1}{2x^2+5x-4}dx =$$

$$\frac{3}{4}\ln\left|2x^2+5x-4\right| + \frac{1}{4}\int \frac{1}{2\left[\left(x+\frac{5}{4}\right)^2-\left(\frac{\sqrt{57}}{4}\right)^2\right]}dx = \frac{3}{4}\ln\left|2x^2+5x-4\right| + \frac{1}{2}\cdot\frac{1}{2\cdot\frac{\sqrt{57}}{4}}\ln\left|\frac{4x+5-\sqrt{57}}{4x+5+\sqrt{57}}\right| + C$$

Să se calculeze:

1. $\int \frac{1}{3x+5}dx$ 2. $\int \frac{2x+3}{2x+1}dx$ 3. $\int \frac{x}{x+4}dx$ 4. $\int \frac{1-3x}{2x+3}dx$

5. $\int \frac{1}{(2x+3)^{2005}}dx$ 6. $\int \frac{1}{x^2-9}dx$ 7. $\int \frac{1}{x^2+4}dx$ 8. $\int \frac{x^2}{x^2-2}dx$

9. $\int \frac{x^2}{x^2+1}dx$ 10. $\int \frac{1}{3x^2+5}dx$ 11. $\int \frac{1}{(x-1)(x-2)}dx$ 12. $\int \frac{1}{(x+1)(x+2)}dx$

13. $\int \frac{1}{x(x+2)}dx$ 14. $\int \frac{1}{x^2-3x+2}dx$ 15. $\int \frac{1}{2x^2-x-3}dx$ 16. $\int \frac{1}{3x^2+x+1}dx$

17. $\int \frac{1}{x^2-2x+5}dx$ 18. $\int \frac{4x-3}{2x^2-3x+1}dx$ 19. $\int \frac{6x-2}{3x^2-2x+5}dx$ 20. $\int \frac{3x-2}{x^2-5x+6}dx$

21. $\int \frac{5x-2}{x^2+4}dx$ 22. $\int \frac{x+1}{x^2+2x+10}dx$ 23. $\int \frac{x^2}{x^6-3}dx$ 24. $\int \frac{x}{x^4+\frac{1}{4}}dx$

25. $\int \frac{2x}{1+x^4}dx$ 26. $\int \frac{x^3}{1+x^8}dx$ 27. $\int \frac{x^3}{(x-1)^{12}}dx$ 28. $\int \frac{x}{(x-1)^{10}}dx$

29. $\int \frac{x^2}{x^6+4}dx$

Rezolvări.

23. Notăm x^3 cu t$\Rightarrow 3x^2dx = dt \Rightarrow$

$$\int \frac{x^2}{x^6-3}dx = \int \frac{1}{t^2-3}\cdot\frac{dt}{3} = \frac{1}{3}\cdot\frac{1}{2\sqrt{3}}\ln\left|\frac{t-\sqrt{3}}{t+\sqrt{3}}\right| + C = \frac{1}{6\sqrt{3}}\ln\left|\frac{x^3-\sqrt{3}}{x^3+\sqrt{3}}\right| + C$$

26. Notăm pe x^4 cu t $\Rightarrow$ 4 x^3 dx=dt $\Rightarrow$

$$\int \frac{x^3}{1+x^8}dx = \int \frac{1}{t^2+1}\cdot\frac{dt}{4} = \frac{1}{4}\cdot arctg\ \ t + C = \frac{1}{4}arctg\ \ x^4 + C$$

27. Notăm pe x-1 cu t $\Rightarrow$ x=t+1 $\Rightarrow$ dx=dt $\Rightarrow$

$$\int \frac{x^3}{(x-1)^{12}}dx=\int \frac{(t+1)^3}{t^{12}}dt=\int \frac{t^3+3t^2+3t+1}{t^{12}}dt=\int (t^{-9}+3t^{-10}+3t^{-11}+t^{-12})dt=\frac{t^{-8}}{-8}+3\frac{t^{-9}}{-9}+3\frac{t^{-10}}{-10}+\frac{t^{-11}}{-11}+C$$

$$= -\frac{1}{8(x-1)^8}-\frac{1}{3(x-1)^9}-\frac{3}{10(x-1)^{10}}-\frac{1}{11(x-1)^{11}}+C$$

28. Notăm pe x-1cu t $\Rightarrow$

$$\int \frac{x}{(x-1)^{10}}dx = \int \frac{t+1}{t^{10}}dt = \int t^{-9}dt + \int t^{-10}dt = \frac{t^{-8}}{-8} + \frac{t^{-9}}{-9} + C = -\frac{1}{8(x-1)^8} - \frac{1}{9(x-1)^9} + C$$

Să se calculeze integralele folosind descompunerea în fracţii raţionale simple.

30. $\int \frac{x-4}{(x-2)\cdot(x-3)}dx$ 31. $\int \frac{1}{(x+2)\cdot(x+5)}dx$ 32. $\int \frac{x^2+5x+7}{x+3}dx$ 33. $\int \frac{(x+1)^3}{x^2-x}dx$

34. $\int \frac{x^3}{x-2}dx$ 34. $\int \frac{x^2+1}{x-1}dx$ 35. $\int \frac{x^4+x+1}{x-1}dx$ 36. $\int \frac{1}{x^2-2x}dx$

37. $\int \frac{1}{x^2+4x}dx$ 38. $\int \frac{x}{x^2-6x+5}dx$ 39. $\int \frac{1}{6x^3-7x^2-3x}dx$ 40. $\int \frac{1}{x(x+1)(x+2)}dx$

41. $\int \frac{x^2+x+2}{(x+1)(x^2+1)}dx$ 42. $\int \frac{x^2-3x+2}{x^3+2x^2+x}dx$ 43. $\int \frac{x^3-2x^2+4}{x^2(x-2)^2}dx$ 44. $\int \frac{1}{x^4-x^2}dx$

45. $\int \frac{x}{(x+1)(x+3)(x+5)}dx$