Soluţie-profil economic

Varianta 4.

Rezolvare:

I. **1**. Condiţii: $x^2-5\geq 0\Rightarrow x\in(-\infty,-\sqrt{5}]\cup[\sqrt{5},+\infty)$.

Avem: $2^{2\left(x-\sqrt{x^2-5}\right)}-12\cdot 2^{x-\sqrt{x^2-5}}\cdot\frac{1}{2}+8=0$. Notăm: $2^{x-\sqrt{x^2-5}}=y$ şi obţinem:

$y^2-6y+8=0$, cu $\Delta=4$, $y_1=2$ şi $y_2=4$. Din: $2^{x-\sqrt{x^2-5}}=2\Rightarrow x-\sqrt{x^2-5}=1\Rightarrow$

$\Rightarrow x-1=\sqrt{x^2-5}$ (cond. $x-1\neq 0\Rightarrow x\neq 1$). Prin ridicare la pătrat, avem:

$x^2-2x+1=x^2-5\Rightarrow -2x=-6\Rightarrow x_1=3$, soluţie.

Din: $2^{x-\sqrt{x^2-5}}=2^2\Rightarrow x-\sqrt{x^2-5}=2\Rightarrow x-2=\sqrt{x^2-5}$ (cond. $x-2\geq 0$, $x\geq 2$). Obţinem:
$x^2-4x+4=x^2-5\Rightarrow -4x=-9\Rightarrow x_2=\frac{9}{2}$, soluţie.

2. Ecuaţia are rădăcini reale strict pozitive dacă: $\Delta>0$, $S=x_1+x_2=-\frac{b}{a}>0$ şi $P=x_1x_2=\frac{c}{a}>0$. Avem: $\Delta=16(1-2m)^2-16m[3(m-1)]=16(m^2-m+1)>0$, $\forall x\in\mathbf{R}$.

$S=-\frac{b}{a}=-\frac{4(1-2m)}{4m}=$

$m\in(-\infty,0)\cup(1/2,+\infty)$

m	$-\infty$		0		1/2		$+\infty$
2m–1	–	–	–	–	0	+	+
m	–	–	0	+	+	+	+
S	+	+	/	–	0	+	+

m	$-\infty$		0		1		$+\infty$
3(m–1)	–	–	–	–	0	+	+

$P=\frac{3(m-1)}{4m}$

$m\in(-\infty,0)\cup(1,+\infty)$

Soluţia: $m\in(-\infty,0)\cup(1,+\infty)$

3. a) $A^2=\begin{pmatrix}2&1&1\\1&2&1\\1&1&2\end{pmatrix}\cdot\begin{pmatrix}2&1&1\\1&2&1\\1&1&2\end{pmatrix}=\begin{pmatrix}6&5&5\\5&6&5\\5&5&6\end{pmatrix}$

$xA+yI_3=\begin{pmatrix} 2x & x & x \\ x & 2x & x \\ x & x & 2x \end{pmatrix}+\begin{pmatrix} y & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & y \end{pmatrix}=\begin{pmatrix} 2x+y & x & x \\ x & 2x+y & x \\ x & x & 2x+y \end{pmatrix}$. Obţinem:

$$\begin{cases} 2x+y=6 \\ x=5 \end{cases} \Rightarrow \begin{cases} x=5\in R \\ y=-4\in R \end{cases}$$

b) d=detA=8+1+1–2–2–2=2≠0⇒A–nesingulară. Atunci: $A^{-1}=\frac{1}{d}A^*$;

$$A_{11}=(-1)^{1+1}=\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}=4-1=3 \quad A_{21}=(-1)^{2+1}\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}=-1 \quad A_{31}=(-1)^{3+1}\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}=-1$$

$$A_{12}=(-1)^{1+2}=\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}=-2+1=-1 \quad A_{22}=(-1)^{2+2}\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}=3 \quad A_{32}=(-1)^{3+2}\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}=-1$$

$$A_{13}=(-1)^{1+3}=\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix}=1-2=-1 \quad A_{23}=(-1)^{2+3}\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}=-1 \quad A_{33}=(-1)^{3+3}\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}=3$$

$$A^*=\begin{pmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{pmatrix} \Rightarrow A^{-1}=\begin{pmatrix} \frac{3}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{3}{2} \end{pmatrix}$$

II. **1**. Ne bazăm pe proprietatea că orice funcţie continuă admite primitive.

Se pune problema continuităţii în punctul x=1.

$$l_s=\lim_{\substack{x\to 1 \\ x<1}} f(x)=\lim_{\substack{x\to 1 \\ x<1}}\frac{x-1}{e^x}=0, \quad l_d=\lim_{\substack{x\to 1 \\ x>1}} f(x)=\lim_{\substack{x\to 1 \\ x>1}}\frac{\ln^2 x}{x}=\frac{\ln^2 1}{1}=0=f(1)$$

$l_s=l_d=f(1)=0\Rightarrow$f continuă în x=1⇒f admite primitive pe **R**.

Calculul primitivei: pentru x∈(–∞,1]

$$\int f(x)dx=-\int(x-1)\left(e^{-x}\right)'dx=-(x-1)e^{-x}+\int e^{-x}dx=e^{-x}(-x+1-1)+C_1=-xe^{-x}+C_1$$

Pentru x∈[1,+∞) $\int f(x)dx=\int \frac{\ln^2 x}{x}dx=\int \ln^2(x)\left(\ln x\right)'dx=\frac{\ln^3 x}{3}+C_2$

Determinăm C_2 în funcţie de C_1 din condiţia ca F să fie aceeaşi în x=1

$F(1)|_{x<1}=F(1)|_{x>1}\Rightarrow -e^{-1}+C_1=0+C_2\Rightarrow C_2=-e^{-1}+C_1$, atunci:

$$F(x)=\begin{cases}-x-e^{-x}+C, pt\ x<1\\ \dfrac{\ln^3 x}{3}-\dfrac{1}{e}+C, pt\ x\geq 1\end{cases}$$

2. Avem: a= –1–b

$$\lim_{n\to\infty}\left[(-1-b)\sqrt{n+1}+b\sqrt{n+2}+\sqrt{n+3}\right]=$$
$$=\lim_{n\to\infty}\left[b\left(\sqrt{n+2}-\sqrt{n+1}\right)+\left(\sqrt{n+4}-\sqrt{n+1}\right)\right]=$$
$$=\lim_{n\to\infty}\frac{n+2-n-1}{\sqrt{n+2}+\sqrt{n+1}}+\lim_{n\to\infty}\frac{n+4-n-1}{\sqrt{n+4}+\sqrt{n+1}}=0+0=0$$

III. Ecuaţia dreptei d: $y-y_0=m_d(x-x_0)$, unde $A(x_0,y_0)$ şi $m_d=-1/m_{AO}$.

$m_{AO}=\dfrac{y_0}{x_0}=\dfrac{2}{1}=2$, atunci $m_d=-\dfrac{1}{2}$ şi obţinem:

d: $y-2=-\dfrac{1}{2}(x-1)\Rightarrow x-2y-5=0$.