Soluţie-matematică-fizică

Varianta 4.

Rezolvare:

I. **1.** Suma coeficienţilor binomiali ai ultimilor trei termeni:

$$C_n^{n-2}+C_n^{n-1}+C_n^{n}=22$$

$$\frac{n!}{(n-2)!\cdot 2!}+\frac{n!}{(n-1)!\cdot 1!}+1=22 \Rightarrow \frac{n(n-1)}{2}+n+1=22$$

$n^2+n-42=0$ care are soluţiile $n_1=-7$ şi $n_2=6$. Din condiţiile ca $n\in\mathbf{N}$ şi $n\neq 2$ obţinem $n=6$.

Avem binomul: $\left(2^{\frac{x}{2}}+2^{\frac{1-x}{2}}\right)^6$. Din formula termenului general $T_{k+1}=C_n^k a^{n-k}b^k$

obţinem: $T_3=T_{2+1}=C_6^2\left(2^{\frac{x}{2}}\right)^4\left(2^{\frac{1-x}{2}}\right)^4=15\cdot 2^{2x}\cdot 2^{1-x}=15\cdot 2^{x+1}$ şi

$T_5=T_{4+1}=C_6^4\left(2^{\frac{x}{2}}\right)^2\left(2^{\frac{1-x}{2}}\right)^4=15\cdot 2^{x}\cdot 2^{2(1-x)}=15\cdot 2^{2-x}$. Ajungem la ecuaţia exponenţială:

$$15\left(2^{x+1}+2^{2-x}\right)=135 \Rightarrow 2^{x+1}+2^{2-x}=9 \Rightarrow 2\cdot 2^x+\frac{4}{2^x}=9 \,\Big|\, 2^x$$

$2\cdot 2^{2x}-9\cdot 2^x+4=0$. Notăm $2^x=y$ şi avem: $2y^2-9y+4=0$ cu soluţiile $y_1=\frac{1}{2}$ şi $y_2=4$. Aflăm necunoscuta x: $2^x=1/2 \Rightarrow x=-1$ şi $2^x=4 \Rightarrow x=2$.

2. Din condiţiile înmulţirii matricelor (m.n)(3.3)=(2.3) obţinem matricea X de forma (2.3).

$X=\begin{pmatrix} x & y & z \\ t & u & w \end{pmatrix}$. Înlocuim şi efectuăm înmulţirea:

$\begin{pmatrix} x & y & z \\ t & u & w \end{pmatrix}\cdot\begin{pmatrix} -3 & 4 & 0 \\ 1 & 1 & -2 \\ -2 & -1 & 3 \end{pmatrix}=\begin{pmatrix} -3x+y-2z & 4x+y-z & -2y+3z \\ -3t+u-2w & 4t+u-w & -2u+3w \end{pmatrix}$. Din egalitatea matricelor obţinem sistemele:

$\begin{cases} -3x+y-2z=2 \\ 4x+y-z=1 \\ -2y+3z=0 \end{cases}$ care este un sistem de tip Cramer.

$$d=\begin{vmatrix}-3 & 1 & -2\\ 4 & 1 & -1\\ 0 & -2 & 3\end{vmatrix}=1,\ d_x=\begin{vmatrix}2 & 1 & -2\\ 1 & 1 & -1\\ 0 & -2 & 3\end{vmatrix}=3,\ d_y=\begin{vmatrix}-3 & 2 & -2\\ 4 & 1 & -1\\ 0 & 0 & 3\end{vmatrix}=-33$$

$$d_z=\begin{vmatrix}-3 & 1 & 2\\ 4 & 1 & 1\\ 0 & -2 & 0\end{vmatrix}=-22,\quad x=\frac{d_x}{d}=3, y=\frac{d_y}{d}=-33,\ z=\frac{d_z}{d}=-22$$

$\begin{cases}-3t+u-2w=-1\\ 4t+u-w=3\\ -2u+3w=-2\end{cases}$ care are soluţiile t=0, u=7 şi w=4.

Matricea X= $\begin{pmatrix}3 & -33 & -22\\ 0 & 7 & 4\end{pmatrix}$.

3. Pentru a rezolva sistemul prin reducerea lui y înmulţim prima ecuaţie cu $\hat{2}$ şi obţinem:

$\begin{cases}\hat{2}x+\hat{4}y=\hat{2}\\ \hat{3}x+\hat{4}y=\hat{1}\end{cases}$ obţinem x=$\hat{7}$. Din prima ecuaţie avem: $\hat{7}+\hat{2}y=\hat{1}$, rezultă

$\hat{5}x \quad / \ =\hat{3}$

$\hat{2}y=\hat{2},\quad y\in\{\hat{1},\hat{5}\}$. Verificare: $\begin{matrix}\hat{3}\cdot\hat{7}+\hat{4}\cdot\hat{1}=\hat{5}+\hat{4}=\hat{1}\\ \hat{3}\cdot\hat{7}+\hat{4}\cdot\hat{5}=\hat{5}+\hat{4}=\hat{1}\end{matrix}$

II. **1.** $\lim\limits_{x\to\infty}\left[\sqrt{2x^2+4x+1}-(ax+b)\right]=\lim\limits_{x\to\infty}\dfrac{2x^2+4x+1-(ax+b)^2}{\sqrt{2x^2+4x+1}+(ax+b)}=$

$$=\lim_{x\to\infty}\frac{2x^2+4x+1-a^2x^2-2abx-b^2}{\sqrt{2x^2+4x+1}+ax+b}=\lim_{x\to\infty}\frac{x^2(2-a^2)+x(4-2ab)+1-b^2}{\sqrt{2x^2+4x+1}+ax+b}$$

Din L=$2\sqrt{2}$ $\Rightarrow$ $2-a^2=0$ $\Rightarrow$ a=$\pm\sqrt{2}$.

Pentru a=$\sqrt{2}$, avem:

$$L=\lim_{x\to\infty}\frac{x(4-2\sqrt{2}b)+1-b^2}{\sqrt{2x^2+4x+1}+\sqrt{2}x+b}=\lim_{x\to\infty}\frac{x\left[4-2\sqrt{2}b+\dfrac{1-b^2}{x}\right]}{x\left(\sqrt{2+\dfrac{4}{x}+\dfrac{1}{x^2}}+\sqrt{2}+\dfrac{b}{x}\right)}=\frac{4-2\sqrt{2}b}{\sqrt{2}+\sqrt{2}}$$

$$\frac{4-2\sqrt{2}b}{2\sqrt{2}}=2\sqrt{2}\Rightarrow 4-2\sqrt{2}b=8\Rightarrow -2\sqrt{2}b=4\Rightarrow b=-\sqrt{2}.$$

Pentru a=$-\sqrt{2}$, avem $\lim\limits_{x\to\infty}\left[\sqrt{2x^2+4x+1}+\sqrt{2}x-b)\right]=+\infty$ $\Rightarrow$ a= $-\sqrt{2}$ nu convine.

2. a) $I_0=\int_0^{\frac{\pi}{4}}\cos 2x dx=\left.\frac{\sin 2x}{2}\right|_0^{\frac{\pi}{4}}=\frac{\sin\frac{\pi}{2}}{2}=\frac{1}{2}$

$$I_1=\int_0^{\frac{\pi}{4}}x\cos 2x dx=x\left.\frac{\sin 2x}{2}\right|_0^{\frac{\pi}{4}}-\frac{1}{2}\int_0^{\frac{\pi}{4}}\sin 2x dx=\frac{\pi}{4}\cdot\frac{1}{2}+\frac{1}{2}\cdot\left.\frac{\cos 2x}{2}\right|_0^{\frac{\pi}{4}}=\frac{1}{2}\left(\frac{\pi}{4}-\frac{1}{2}\right)$$

b) $I_n-I_{n+1}=\int_0^{\frac{\pi}{4}}\left(x^n\cos 2x-x^{n+1}\cos 2x\right)dx=\int_0^{\frac{\pi}{4}}(1-x)x^n\cos 2x dx\ge 0\Rightarrow(I_n)_{n\in\mathbf{N}}$ şir monoton crescător.

c) $(J_n)_{n\in\mathbf{N}}$ este monoton descrescător cu limita 0.

$$I_n-J_n=\int_0^{\frac{\pi}{4}}x^n(\cos 2x-1)dx\le 0\Rightarrow I_n\le J_n\Rightarrow I_n\to 0, n\to\infty$$

III. Calculăm pantele dreptelor AB şi AC: $m=\frac{y_2-y_1}{x_2-x_1}$

$$\left.\begin{array}{l} m_{AB}=\frac{1-5}{5-2}=-\frac{4}{3} \\ m_{AC}=\frac{2-5}{-2-2}=\frac{3}{4}\end{array}\right\}\Rightarrow m_{AB}=-\frac{1}{m_{AC}}\Rightarrow AB\perp AC\Rightarrow \Delta ABC \text{ este dreptunghic} \Rightarrow \text{centrul}$$

cercului circumscris este la mijlocul [BC].

M_0: $x_0=\frac{x_1+x_2}{2}=\frac{3}{2},\quad y_0=\frac{y_1+y_2}{2}=\frac{3}{2}$

$$r=\frac{BC}{2}\Rightarrow r^2=\frac{BC^2}{4}=\frac{(x_2-x_1)^2+(y_2-y_1)^2}{4}=\frac{(5+2)^2+(1-2)^2}{4}=\frac{50}{4}$$

$$C:(x-x_0)^2+(y-y_0)^2=r^2\Rightarrow\left(x-\frac{3}{2}\right)^2+\left(y-\frac{3}{2}\right)^2=\frac{50}{4}$$