Logaritmi -exerciții rezolvate

1) Să se calculeze:

$$2\log_5 \sqrt[4]{5}+\frac{1}{2}\log_{\sqrt{5}} 25-\log_5^2 \sqrt{5}-2=$$

$$=2\cdot\frac{1}{4}+\log_{\sqrt{5}} 5-\left(\frac{1}{2}\log_5 5\right)^2-2=\frac{1}{2}+\log_{\sqrt{5}}\left(\sqrt{5}\right)^2-\frac{1}{4}-2=\frac{1}{2}+2-\frac{1}{4}-2=\frac{1}{4}.$$

2) Știind că $\log_2 5=a,\log_2 3=b$ să se calculeze:

$$\log_8 75=\log_8(25\cdot 3)=\log_8 25+\log_8 3=2\cdot\frac{1}{\log_5 8}+\frac{1}{\log_3 8}=\frac{2}{3\log_5 2}+\frac{1}{3\log_3 2}=$$

$$=\frac{2}{3}\log_2 5+\frac{1}{3}\log_2 3=\frac{2a+b}{3}$$

3) Știind că $\log_2 5=a,\log_2 3=b$ să se calculeze:

$$\log_{15} 12=\log_{15} 2^2\cdot 3=2\log_{15} 2+\log_{15} 3==\frac{2}{\log_2 15}+\frac{1}{\log_3 15}=$$

$$=\frac{2}{\log_2 3+\log_2 5}+\frac{1}{\log_3 3+\log_3 5}=\frac{2}{b+a}+\frac{1}{1+\frac{\log_2 5}{\log_2 3}}=\frac{2+b}{a+b}$$

4) Să se calculeze :

$$\log_4\left[\log_3\left(\log_{\frac{1}{2}}\frac{1}{512}\right)\right]=\log_4\left[\log_3\left(\log_{\frac{1}{2}}\left(\frac{1}{2}\right)^9\right)\right]=\log_4(\log_3 9)=\log_4 2=\log_4 4^{\frac{1}{2}}=\frac{1}{2}.$$

5) Să se calculeze : $\log_{\frac{2}{3}}\left(\log_5 3^{\log_3\sqrt[3]{25}}\right)=\log_{\frac{2}{3}}\left(\log_5 5^{\frac{2}{3}}\right)=\log_{\frac{2}{3}}\frac{2}{3}=1.$

6) Să se calculeze :

$$\log_{\frac{1}{2}}\left[\log_{\sqrt{6}}\left(\log_3 729\right)\right]=\log_{\frac{1}{2}}\left[\log_{\sqrt{6}}\left(\log_3 3^6\right)\right]=\log_{\frac{1}{2}}\left[\log_{\sqrt{6}} 6\right]=\log_{\frac{1}{2}}\frac{1}{2}=1.$$

7) Să se arate că următoarea expresie este independentă de x:

$$A=\frac{\log_5 x^2+\log_5 x^3}{\log_4 x^2+\log_4 x^3}=\frac{\log_5 x^5}{\log_4 x^5}=\frac{\log_5 x}{\log_4 x}=\frac{\frac{\lg x}{\lg 5}}{\frac{\lg x}{\lg 4}}=\frac{\lg 4}{\lg 5}=\log_5 4=ct.$$

8) Demonstraţi egalitatea: $\log_3 2\cdot\log_4 3\cdot\log_5 4\cdot\log_6 5\cdot\log_7 6\cdot\log_8 7=\frac{1}{3}$

$$\frac{\lg 2}{\lg 3}\cdot\frac{\lg 3}{\lg 4}\cdot\frac{\lg 4}{\lg 5}\cdot\frac{\lg 5}{\lg 6}\cdot\frac{\lg 6}{\lg 7}\cdot\frac{\lg 7}{\lg 8}=\frac{\lg 2}{\lg 8}=\frac{\lg 2}{3\lg 2}=\frac{1}{3}.$$

9) Folosind regulile de calcul cu logaritmi, să se scrie într-o formă mai simplă:

$$\log_7 35 + \log_7 9 - \log_7 45 = \log_7 \frac{35 \cdot 9}{45} = \log_7 7 = 1 .$$

10) Folosind regulile de calcul cu logaritmi, să se scrie într-o formă mai simplă:

$$E = \log_{13} \sqrt[3]{169} + \log_{13} \sqrt{4+\sqrt{3}} + \log_{13} \sqrt{3+\sqrt{5+\sqrt{3}}} + \log_{13} \sqrt{3-\sqrt{5+\sqrt{3}}} .$$

$$E = \log_{13} \sqrt[3]{169} + \log_{13}(\sqrt{4+\sqrt{3}} \cdot \sqrt{3+\sqrt{5+\sqrt{3}}} \cdot \sqrt{3-\sqrt{5+\sqrt{3}}}) =$$

$$= \log_{13} \sqrt[3]{169} + \log_{13}(\sqrt{4+\sqrt{3}} \cdot \sqrt{4-\sqrt{3})} = \log_{13} \sqrt[3]{169} + \log_{13} \sqrt{13} = \log_{13}(13^{\frac{2}{3}} \cdot 13^{\frac{1}{2}}) = \frac{7}{6}$$

11) Rezultatul calculului $\log_2(\lg 10^4) - \log_8 384 + \log_8 3 - \log_{\frac{1}{8}} \sqrt[4]{9\sqrt[8]{243}}$ este

a) $\frac{41}{12}$; b) $\frac{7}{12}$; c) 1; d) $-\frac{5}{3}$.

$$E = \log_2 4 - \log_8 2^7 \cdot 3 + \log_8 3 - \log_{\frac{1}{3}} 3^{\frac{11}{12}} = 2 + \log_8 2^{-7} - \log_{\frac{1}{3}} (\frac{1}{3})^{\frac{-11}{12}} = 2 + \frac{11}{12} - 7\log_8 2 =$$

$$= \frac{35}{12} - 7 \cdot \frac{1}{\log_2 8} = \frac{35}{12} - \frac{7}{3} = \frac{7}{12}$$

Deci, varianta corectă este b).